20. **Numerical experiments.** Write a short program that performs weighted Jacobi (with variable ω), Gauss–Seidel, and red-black Gauss–Seidel for the one-dimensional model problem. First reproduce the experiments shown in Fig. 2.3. Then experiment with initial guesses with different wavenumbers. Describe how each method performs as the wavenumbers increase and approach n.

13. V-Cycle program. Develop a V-cycle program for the one-dimensional model problem. Write a subroutine for each individual component of the algorithm as follows.

(a) Given an approximation array **v**, a right-side array **f**, and a level number $1 \le l \le L$, write a subroutine that will carry out ν weighted Jacobi sweeps on level l.

(b) Given an array **f** and a level number $1 \le l \le L-1$, write a subroutine that will carry out full weighting between level l and level $l+1$.

(c) Given an array **v** and a level number $2 \le l \le L$, write a subroutine that will carry out linear interpolation between level l and level $l-1$.

(d) Write a driver program that initializes the data arrays and carries out a V-cycle by calling the three preceding subroutines. The program should be tested on simple problems for which the exact solution is known. For example, for fixed k, take $f(x) = C\sin(k\pi x)$ on the interval $0 \le x \le 1$, where C is a constant. Then

$$u(x) = \frac{C}{\pi^2 k^2 + \sigma}\sin(k\pi x)$$

is an exact solution to model problem (1.1). Another subroutine that computes norms of errors and residuals will be useful.