研究生《复分析》考试卷子//每题25分

2024秋季

1. Let μ be the modular function on $\Delta = \{|z| < 1\}$ defined by Riemann map and reflections. To prove:

a) it gives a covering map from Δ to $\mathbb{C} \setminus \{0, 1\}$ by μ;

b) the induced Poincare metric $\rho_{0,1}$ on $\mathbb{C} \setminus \{0, 1\}$ satisfies

$$\lim_{z \to 0} \frac{|z|\rho_{0,1}(z)}{\ln |z|^{-1}} = 1.$$

2. Please give a conformal structure on a torus surface T^2 and then to show there is a smooth curve S on T^2 such that the character 1-form via S defines a non-trivial cohomology class of differential 1-forms.

3. a) To state the Riemann-Roch theorem on a compact Riemannian surface M.

b) To prove by the Riemann-Roch theorem that the dimension of space of holomorphic 2-multiple differentials is $3(g-1)$, where $g > 1$ is the genus of M.

4. a) To define the modula space $\mathcal{M}(S_0)$ and Teichmuller space $T(S_0)$ on a compact Riemannian surface S_0.

b) Let σ^* be the modula group on $T(S_0)$. To prove σ^* is an isometric group with respect to the Teichmuller distance $d_T(\cdot, \cdot)$ on $T(S_0)$.